Student's Name

Instructor's Name

Course

Date

Partial Differential Equation and Their Application in Heat Transfer

Introduction

A differential equation associates a function's derivative with its variables. Fundamentally, there are two types of differential equations: ordinary and Partial Differential Equations. A function that depends on a single variable is an ordinary differential equation, while a Partial Differential Equation (PDE) is a function that depends on more than one variable (Olver 1). Partial Differential Equations (PDE) are essential in developing models critical in understanding and implementing the most fundamental engineering and physics theories. Partial differential equations have a wide application in various fields, including fluid mechanics, electricity, magnetism, aerospace, communication, etc.

Brief Overview of PDE

A PDE equation has one or more functions with their corresponding partial derivatives. Below is a sample denotation of a PDE.

$$\frac{\partial_u}{\partial_t} = \frac{\partial^2 u}{\partial x^2} Equation 1$$

In the above equation, u is a function that has two variables, x, and t.

The solution to partial differential equations can be determined either numerically or analytically. Analytical methods of solving PDE include using Laplace transforms, integral transform techniques, and Fourier transforms (Borthwick 1). Numerical methods use software simulations to approximate a solution to a PDE problem.

Application of PDE in Heat Transfer

Consider a metallic bar. The heat transfer in the bar is determined by its physical properties, specifically, its specific heat σ and density, ρ. Therefore, the heat transfer from point x_1 to point x_2 in time t can be represented by the following derivative.

$$D_{x_1,x_2} = \int_{x_1}^{x_2} \rho(x)\sigma(x)u(x,t)dx Equation 2$$

Therefore, to determine the rate of heat transfer from point x_1 to x_2, the integral of equation 2 is determined, as indicated below.

$$\frac{d}{dt}\left[\int_{x_1}^{x_2} \rho(x)\sigma(x)u(x,t)dx\right] = \int_{x_1}^{x_2}\int_{x_1}^{x_2} \rho(x)\sigma(x)\frac{\partial u}{\partial t}(x,t)dx Equation 3$$

If the principle of heat conservation is applied, it implies that at this point, x_1, the heat leaving is equal to the heat entering the point, as indicated below.

$$D_{x_1,x_2}(heating leaving the point) =- \int_{x_1}^{x_2} \rho(x)\sigma(x)u(x,t)dx Equation 4$$

If the rate of heat generation per unit volume is given by the function $\mu(x)u(x,t) + \nu(x)$ and the flow of heat in the metal bar is constant, then the rate at which the heat is leaving the point x_1 is as indicated below.

$$-\int_{x_1}^{x_2} \rho(x)\sigma(x)u(x,t)dx + \int_{x_1}^{x_2} \mu(x)u(x,t) + \nu(x)dx Equation 5$$

Also, it is known that the rate of heat flow is related to the temperature change. Therefore, for a given time, t, and at position x, the heat flow rate is modelled using the following PDE.

$$F(x,t) =- k(x)\frac{\partial u}{\partial x}(x,t) Equation 6$$

In the above equation, the term $k(x)$ represents the thermal conductivity of the metal bar at the position x. Simplifying equation 6 results in an equation that shows the rate at which the heat leaves the position x_1 to x_2.

$$F\left(x_2, t\right) - F\left(x_t t\right) = \int_{x_1}^{x_2} \frac{\partial F}{\partial x}(x, t)dx Equation 7$$

Both Equation 7 and 4 shows the rate of heat leaving a given point. Therefore, the two equations are equal.

$$\int_{x_1}^{x_2} \frac{\partial F}{\partial x}(x, t)dx =- \int_{x_1}^{x_2} \rho(x)\sigma(x)u(x, t)dx Equation 8$$

Equation 8 holds for any points on the metallic bar. Therefore, since the integrands on both sides of Equation 8 are equal, the heat flow can be modelled by the following equation.

$$\int_{x_1}^{x_2} \rho(x)\sigma(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right) + \mu(x)u + \nu(x). Equation 9$$

Equation 9 can be used to model heat transfer in both simple and complex cases. The equation can be used to determine at any point in a solid structure.

Conclusion

Partial differential equations have numerous applications in engineering and physics. PDEs can be used to simplify and model complex problems. In this paper, a partial differential equation has been used to develop a function that can be used to model heat transfer at any point in solid structure.

Works Cited

Borthwick, David. *Introduction to Partial Differential Equations*. Springer, 2017.

Olver, Peter. *Introduction to Partial Differential Equations*. Springer Science & Business Media, 2013.